# Housing Sector-specific DSGE Model with Applications to Czech and Slovak Economies<sup>1</sup>

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## Abstract

In this paper, we introduce and use a dynamic stochastic general equilibrium (DSGE) model tailored for analysis of small open economies, which is further amended to encompass housing sector-specific dynamics and to generate relevant insight and conditional forecasts for the housing sector. We analyse and compare the housing sector dynamic behaviour for the Czech Republic and the Slovak Republic. The empirical part of our paper consists of Bayesian estimation and evaluation of the model, impulse response analysis and conditional forecasts under alternative macroeconomic policy scenarios. We find significant pro-volatile impact of higher loan to values (LTVs) for both economies analysed. This effect is observed both in IRFs and conditional forecasts calculated using different LTV-based scenarios.

**Keywords:** DSGE model, housing sector, conditional forecasts, loan to value (LTV)

JEL Classification: C11, C51, E17, R39

# 1. Introduction

During the last three decades, many variants and iterations of dynamic stochastic general equilibrium (DSGE) models were built and Bayesian estimation has gained ground over classical methods in this field. Nowadays, many types of DSGE models based on explicit microeconomic foundations are able to generate data series that resemble key macroeconomic variables remarkably well, both for large economic entities and for small open economies (SOE) such as the Czech Republic (CR) and the Slovak Republic (SR). Usually, DSGE models are tailored

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to serve particular purposes. Starting from some generally designed model, specific equations and equation groups may be added or extended for a better delineation of a desired topic, such as output gap dynamics, monetary policy transmission mechanisms and for other sector-specific analyses and forecasts.



Graph 1 Year-on-year % Changes in Real House Prices and Private Consumption

Source: Authors' own calculations based on data from the Czech and Slovak statistical offices.

The 2008 crisis and its consequences have shown the cardinal underestimation of proper and accurate monitoring and modelling of the dynamic interactions of the U.S. housing sector. Although the overall conditions in pre-crisis U.S economy differ substantially from the current states of today's Czech and Slovak economies, the underlying relations between housing sector and consumption remain a substantial factor. As an example, the compound Graph 1 points out the strong and positive correlation between the year-on-year (y-o-y) percentage changes in private consumption and real house price changes (nominal house inflation/deflation corrected by consumer price index (CPI) inflation). For both economies, by comparing the scales on left and right y-axes, we may see a strong correlation between house prices and private consumption. Also, it may be observed that house-price fluctuations have a much wider amplitude, which is consistent with their usual cycle enhancing interpretation.

The first goal of our paper is to draw public attention to the dynamics, transmission mechanisms and the role that housing sector plays in SOEs such as the Czech Republic and the Slovak Republic. Hence, we introduce and describe a substantially improved version of a DSGE model for SOEs such as CR and SR, with elaborated housing sector. We use the estimated model for application purposes. Specifically, observable data for CR and SR are combined with prior information and used to estimate model parameters and to generate alternative (scenario-based) impulse response functions (IRFs) and forecasts. Two key housing sector-specific features of our model deserve distinctive mentioning here:

*Loan to Value* (LTV) ratio: *individual* LTV = mortgage/value is the ratio of mortgage amount taken against the appraised value of a given property. At the *macroeconomic level*, high overall housing sector LTV is usually perceived as pro-cyclical and consumption enhancing and as a risk factor. Higher overall LTV ratio would support GDP growth during expansion periods, but it may also augment the severity of business cycle downturns. Therefore, in our paper we aim to investigate the properties of macroeconomic dynamics for Czech and Slovak economies under alternatively set (calibrated) LTVs. Specifically, we analyse alternative macroeconomic LTV settings, simulating both macroprudential measures (the expected macroeconomic impact of stricter individual LTV rules being imposed) and possible expansionary politics focused on the housing sector (i.e. measures and sectorial incentives that would result in increased mortgage exposure of the households). Such approach may serve as a basis for selection and evaluation of future economic policies that may be imposed by relevant central authorities in order to mitigate financial and overall instabilities experienced during different phases of the business cycle.

**Real house-price** (q) dynamics as shown in Graphs 1 and 2 is calculated as follows: y-o-y relative differences of nominal housing sector prices (*HP*) are corrected by CPI inflation and expressed in percentage points. The construction and dynamics of q is shown in Graph 2.

This paper is organized as follows: Sections 2 and 3 contain the theoretical background and description of the DSGE model used. Section 4 deals with Bayesian estimation topics and IRFs. Section 5 is dedicated to applications arising from the model and last section concludes.

## Graph 2

Changes in Real House Prices (q), CPI and Nominal House-Prices (HP) for CR and SR



Source: Authors' own calculations based on data from the Czech and Slovak statistical offices.

# 2. Brief Literature Review

DSGE models are a popular and well established research tool for many applications, such as economic policy analysis, forecasting, contrafactual analysis and other areas. In this chapter, we only aim to set up a working framework for our SOE DSGE model. For detailed discussion and an extensive list of DSGE--related resources, please refer to Galí and Gertler (2007).

Since the 2008 financial crisis, many DSGE models were often criticized for missing a well-developed housing sector. Nevertheless, Iacoviello and Neri (2010) include land as an explicit factor and focus on the volatility enhancing nature of the housing sector in the USA.

The *Adjusted Present Value (APV) model* introduced by Aoki, Proudman and Vlieghe (2002) is a *closed economy model* with a housing sector. It incorporates heterogeneous households as its most important feature and applies the concept of financial accelerator to the housing market and household consumption in a closed economy environment.

Also, the 2008 crisis and its consequences led to debates over possible monetary policy actions aimed to promote financial and overall stability by imposing LTV-based collateral restrictions at the *individual* level. For example, the Swedish Central Bank has set a maximum LTV of 0.85 on new individual mortgage contracts in 2010. A thorough DSGE-based analysis of the impacts of this macroprudential remedy is provided by Walentin (2013) who analyses the corresponding effects to monetary transmission mechanisms.

Our approach extends the APV model into a SOE DSGE model, providing an environment suitable for analysis of Czech and Slovak economies. Also, we expand on the work of Walentin (2013) by focusing on the LTV-related aspects of macroeconomic dynamics.

## 3. SOE DSGE Model for Housing Sector Analysis

We use the APV model as the basis for constructing our SOE DSGE model (1) - (29). Our extensions and modifications to the APV model are based on incorporating key open economy features (given the SOE nature of CR and SR) and a government sector. This provides substantial improvement in model performance for the two SOEs we aim to analyse: the Czech Republic and the Slovak Republic. The most important open economy features added to our model are as follows: The production function now incorporates imported intermediate goods. The goods producing sector sells to foreign consumers in addition to home consumers. Also, a set proportion of consumers is able to access foreign

capital markets. The model includes households (consumers), firms, central bank and government.

Following from a key APV model feature (see also Campbell and Mankiw, 1989) *there are two types of households* (consumers): One type is able to access the capital markets and can smooth consumption across time by buying or selling financial assets. Such households follow the permanent income hypothesis (PIH). The other type of household follows the rule of thumb (ROT) consumption, spending all their income on consumption. ROT consumers are fully credit-constrained and do not have access to the credit markets.

Keeping in mind the goals of our study and for the sake of simplicity, we adhere to many of the APV model simplifications, such as abstracting from productive capital and international trade in services. However, we decided to diverge from the APV model in one of its important features: we restrict the ROT households from owning any housing assets, as households without access to credit market would be unable to purchase property (in practical terms: mortgage would not be granted). As ROT consumers have no collateral for the mortgage, not to mention their lack of funds for a deposit, this approach seems more realistic than envisaging ROT consumers repeatedly accessing the mortgage market. Each period, PIH and ROT consumers purchase goods from firms, receive wages from labour supplied to firms and pay rent to homeowners.

PIH households are divided into two complementary components: a homeowner and a consumer. The homeowner transacts in the housing market each period, selling the housing stock and purchasing the stock anew. The homeowners borrow against the net worth of their housing stock to meet any shortfall between the price of the housing stock bought at the end of the period and the price realized on sale of the existing housing stock. The net worth of housing is defined as the value of the housing stock less outstanding debt and less any dividends paid to consumers. Homeowners also charge a rental fee to consumers. Thus the housing stock is completely owned by the PIH consumers and the ROT consumers pay rental to their PIH landlords.

Firms are monopolistically competitive and produce a continuum of consumer goods. At each period, firms hire labour from households and purchase intermediate input from home and abroad. Imports are used-up each period and capital is assumed to be constant. The output of firms may be consumed by household or government, exported or used to produce additional housing stock. The conversion of consumer goods to housing stock follows from the APV model. Calvo-type price stickiness applies (see Calvo, 1983). The monetary authority adheres to a Taylor rule reaction function (with lagged inflation and output gap as indicators of inflationary pressure) and uses nominal interest rate as its lever, subject to a smoothing parameter.

# Figure 1 SOE DSGE Dynamics Scheme



Source: Self-prepared.

The government collects lump sum taxes from consumers and purchases consumer goods. For any given period, the government debt is fully funded through the sale of government bonds and if taxes exceed expenditures, the surplus is used to retire debt. For simplicity reasons, government expenditures do not impact households directly (no transfers). Instead, the government acts as a source of final demand for consumer goods (therefore inducing labour demand and imports). Following from Galí and Gertler (2007), fiscal policy is modelled as the combination of exogenous government spending, government debt and lump sum taxes.

Although most basic interactions of the model may be outlined as in Figure 1, a *technical description* is necessary to properly tackle actual model dynamics. Our DSGE model may be described by a system of log-linearized equations (1) to (29). The first equation represents a Cobb-Douglas production function under the assumption of fixed capital. Input demand is determined by (2) and (3) represents the labour market equilibrium. Resource constraint is defined by (4) and (5) represents the export demand. Equations (6) – (11) describe consumption: (6) is a consumption identity, (7) and (8) are the demand equations for consumption goods and housing stock, consumption of PIH and ROT consumers follows (9) and (10) respectively and (11) is the aggregate consumption.

Equation (12) is the equilibrium condition for domestic and foreign investments of financial assets and wages of PIH consumers are given by (13) whereas the wages of ROT consumers have been already described by equation (10) as, by definition, ROT consumers spend all their income on consumption. Equations (14) - (19) describe the housing sector: the dynamics of housing investment demand is described by equations (14) - (16), the net worth of housing investments dynamics depends on the net return from housing investment minus dividend payments as per equation (17), (18) is the dividend rule definition and (19) represents the accumulation of housing capital. The resource constraint for all agents in our model is given by (20). Equation (21) is a practical modification to the New Keynesian Phillips Curve and the output follows (22) under flexible prices assumption. The exchange rate identity is given by (23) and the overall inflation is calculated from (24).

Equation (25) defines the nominal interest rate and (26) describes a monetary policy (Taylor) rule that has been enhanced by inserting an interest rate smoothing parameter. The government debt is driven by (27) and (28) is a fiscal rule determining how expenditures are funded. Finally, equation (29) corresponds to GDP. For *description of all variables and parameters* in the model, please refer to Tables 1 to 3. All variables and parameters are *domestic*, unless stated otherwise. Any hat-labelled variable describes a relative deviation (in %) from a steady state. Variables without a time subscript (*t*) denote the steady state values. Detailed *technical appendix* (TA) is provided.<sup>2</sup>

$$\widehat{Y}_{t} = \varphi \widehat{IM}_{t} + (1 - \varphi) \Big( \widehat{A}_{t} + \widehat{L}_{t} \Big)$$
(1)

$$\widehat{IM}_{t} = \frac{1}{1 - \gamma} \left( \widehat{w}_{t} - \widehat{RS}_{t} - \gamma \widehat{A}_{t} \right) + \widehat{L}_{t}$$
(2)

$$\widehat{mc}_{t} = \widehat{w}_{t} - (1 - \gamma)\varphi(\widehat{IM}_{t} - \widehat{L}_{t})\left[(1 - \gamma)(1 - \varphi) + \gamma\right]\widehat{A}_{t}$$
(3)

$$\hat{Y}_t = \frac{c}{Y}\hat{c}_t + \frac{I}{Y}\hat{I}_t + \frac{G}{Y}\hat{G}_t + \frac{EX}{Y}\widehat{EX}_t$$
(4)

$$\widehat{EX}_{t} = \vartheta \widehat{RS}_{t} + \varsigma \widehat{Y}_{t}^{f}$$
(5)

$$\widehat{X}_{c,t} = -\widehat{X}_{h,t} \left(\frac{1-\upsilon}{\upsilon}\right) \left(\frac{X_h}{X_c}\right)^{1-\eta}$$
(6)

$$\hat{c}_t = \hat{C}_t - \eta \hat{X}_{c,t} \tag{7}$$

$$\hat{h}_t = \hat{C}_t - \eta \widehat{X}_{h,t} \tag{8}$$

<sup>&</sup>lt;sup>2</sup> Technical appendix with derivation and log-linearization of the model (1) - (29), annotated Dynare code, observed data, parameter calibration details, priors, supplementary estimation outputs, graphs, etc. is available from <a href="http://sites.google.com/site/econometricsvse/wps">http://sites.google.com/site/econometricsvse/wps</a>.

$$\widehat{\boldsymbol{C}}_{t}^{p} = \boldsymbol{E}_{t} \widehat{\boldsymbol{C}}_{t+1}^{f} - \widehat{\boldsymbol{R}}_{t+1} \tag{9}$$

$$\widehat{C}_{t}^{r} = \frac{wL^{r}}{C^{r}} \left( \widehat{L}_{t}^{r} + \widehat{w}_{t} \right) - \frac{(1-n)T}{C^{r}} \widehat{T}_{t}$$

$$\tag{10}$$

$$\widehat{C}_t = n_C \widehat{C}_t^P + (1 - n_C) \widehat{C}_t^r \tag{11}$$

$$\widehat{R}_{t+1} = \widehat{R}_{t+1}^f - \delta_b \widehat{b}_t + E_t \widehat{RS}_{t+1} - \widehat{RS}_t$$
(12)

$$\hat{C}_{t}^{p} = \hat{w}_{t} - \frac{L^{p}}{1 - L^{p}} \hat{L}_{t}^{p}$$
(13)

$$E_t \hat{R}_{h,t+1} = \Omega(\hat{N}_{t+1} - \hat{q}_t - \hat{h}_{t+1}) + \hat{R}_{t+1}$$
(14)

$$\widehat{R}_{h,t} = (1-\mu)\widehat{X}_{h,t} + \mu \widehat{q}_t - \widehat{q}_{t-1}$$
(15)

$$\hat{q}_t = \Gamma_d \left( \hat{I}_t - \hat{h}_t \right) + \hat{X}_{c,t} \tag{16}$$

$$\widehat{N}_{t+1} = R_h \Big[ (1+bn) \widehat{R}_{h,t} - bn \widehat{R}_t + (1-bn\Omega) \widehat{N}_t + bn\Omega (\widehat{q}_{t-1} + \widehat{h}_t) \Big] - (R_h - 1) \widehat{D}_t \quad (17)$$

$$\widehat{D}_{t} = \frac{\chi'(\phi)}{\chi(\phi)} \phi \left( \widehat{N}_{t+1} - \widehat{q}_{t} - \widehat{h}_{t+1} \right)$$
(18)

$$\hat{h}_{t+1} = (1-\delta)\hat{h}_t + \delta\hat{I}_t \tag{19}$$

$$\beta C \left[ b_t \left( 1 + a\delta \right) - a\widehat{R}_{t+1}^f \right] = b_{t-1}C + b(\Delta \widehat{S}_t + \widehat{\pi}_t^f - \widehat{\pi}_t) - EX(\varsigma \widehat{Y}_t^f + \vartheta \widehat{RS}_t) + \frac{SP^f}{P} IM(\widehat{IM}_t + \widehat{RS}_t)$$
(20)

$$\hat{\pi}_{c,t} = \kappa \widehat{mc}_t + \beta E_t \hat{\pi}_{c,t+1}$$
(21)

$$\hat{Y}_{t}^{flex} = \frac{-\alpha \left(\frac{IM}{Y}\right)^{\gamma} \widehat{RS}_{t}}{\left(1-\gamma\right) \left[\left(1-\alpha\right) \left(\frac{AL}{Y}\right)^{\gamma} - \left(\frac{1-\gamma}{1-\gamma+\tau}\right)\right]} + \frac{(1-\alpha) \left(\frac{1-\gamma}{1-\gamma+\tau}\right) \left(\frac{AL}{Y}\right)^{\gamma}}{\left(1-\alpha\right) \left(\frac{AL}{Y}\right)^{\gamma} - \left(\frac{1-\gamma}{1-\gamma+\tau}\right)} \widehat{A}_{t} \quad (22)$$

$$E_{t} \widehat{RS}_{t+1} - \widehat{RS}_{t} = E_{t} \widehat{S}_{t+1} - \widehat{S}_{t} + E_{t} \widehat{\pi}_{t+1}^{f} - E_{t} \widehat{\pi}_{t+1} \quad (23)$$

$$\hat{\pi}_{t} - \hat{\pi}_{c,t} = \hat{X}_{c,t-1} - \hat{X}_{c,t}$$
(24)

$$\hat{R}_{t+1}^{n} = \hat{R}_{t+1} + E_t \hat{\pi}_{t+1}$$
(25)

$$\hat{R}_{t+1}^{n} = \rho_{i}\hat{R}_{t}^{n} + (1-\rho_{i})\gamma_{\pi}\hat{\pi}_{t-1} + (1-\rho_{i})\gamma_{y}\tilde{y}_{t} + X_{ii,t}$$
(26)

$$B_t^G = RB_{t-1}^G + R\frac{G}{Y}\hat{G}_t - R\frac{G}{Y}\hat{T}_t$$
(27)

$$\widehat{T}_{t} = \left(\frac{G}{Y}\right)^{-1} \phi_{B} \frac{B_{t}^{G}}{Y} + \phi_{G} \widehat{G}_{t}$$
(28)

$$\hat{y}_t = \frac{Y}{y}\hat{Y}_t - \frac{IM}{y}\widehat{IM}_t$$
(29)

where

$$n_{C} = n \frac{C^{p}}{C}, \ \varphi = \frac{\alpha IM^{\gamma}}{\alpha IM^{\gamma} + (1 - \alpha)(AL)^{\gamma}}, \ \mu = \frac{(1 - \delta)q}{X_{h} + (1 - \delta)q}, \ bn = \frac{1}{\phi} - 1, \ \tau = \frac{L}{1 - L},$$
$$\kappa = \frac{(1 - \theta)(1 - \theta\beta)}{\theta}, \ \phi_{D} = \frac{\chi'(\phi)}{\chi(\phi)}\phi \text{ and } \tilde{y}_{t} = \hat{Y}_{t} - \hat{Y}_{t}^{flex}$$

The autoregressive nature of selected variables and stochastic shocks to the model may be observed from a separate set of equations: (30a) contains a technology shock  $\epsilon_A$ , domestic interest rate shock  $\epsilon_{X_{ii}}$  is in (30b) and foreign real interest rate shock  $\epsilon_{R^f}$  is in (30c). Equation (30d) encompasses foreign demand shock  $\epsilon_{Y^f}$  and the government spending shock  $\epsilon_G$  may be observed in (30e):

$$\hat{A}_t = \rho_A \hat{A}_{t-1} + \epsilon_A \tag{30a}$$

$$X_{ii,t} = \rho_{X_{ii}} X_{ii,t-1} + \epsilon_{X_{ii}}$$
(30b)

$$\hat{R}_{t}^{f} = \rho_{R^{f}} \hat{R}_{t-1}^{f} + \epsilon_{R^{f}}$$
(30c)

$$\hat{Y}_t^f = \rho_{Y^f} \hat{Y}_{t-1}^f + \epsilon_{Y^f}$$
(30d)

$$\hat{G}_t = \rho_G \hat{G}_{t-1} + \epsilon_G \tag{30e}$$

# 4. Bayesian Estimation

Bayesian estimation approach combines the available *observed data* series and relevant *prior knowledge* in order to generate the so-called *posterior estimates* through maximum likelihood estimation (MLE) process. For DSGE models, prior information usually is expressed in the form of calibrated (fixed) parameters and probability distributions of parameters and shocks ascribed to the model. Transparent and justified prior specification is crucial for reliable and credible interpretation of the results. For the purpose of basic model evaluation, parameter priors may be confronted with their posterior distributions from the estimated model. Similarly, observed data moments may be compared with business cycle properties (moments) of the model.

## 4.1. Observed Data, Parameter Calibration and Priors

For each economy, five observed data series (2006Q1 – 2013Q2) are used for the Bayesian estimation of our DSGE model: real gross domestic product *Y*, CPI inflation  $\pi$ , exports *EX* and real house-prices *q* are expressed in terms of relative (%) deviations from a steady state. For example:  $\hat{q}_t = 100(q_t - q_t^*)/q_t^*$ , where the unobservable steady state  $q_t^*$  may be efficiently approximated by a trend component of the Hodrick and Prescott (1997) filter.

Variable	Description	Variable	
С	Aggregate consumption	$C^{f}$	Foreign aggregate consumption
9	Real house price	IM	Imports
Ī	Housing investment	$R_h$	Return on housing
h	Housing stock	D	Housing dividend
$X_c$	Relative price of consumption	EX	Net exports
$Y, \tilde{Y}$	Real output, output gap	RS	Real exchange rate
y	GDP (Y – IM)	Y <sup>f</sup>	Foreign output
Y <sup>flex</sup>	Flexible price output	$R^{f}$	Foreign interest rate
$R^n, R$	Nominal interest rate, Real domestic	,	The borrowing undertaken to finance
	interest rate	В	the purchase of housing stock
Α	Technology	$B^G$	Government debt
L	Aggregate labor	$X_h$	Relative price of renting
w	Real wage	G	Government spending
mc	Real marginal cost	Т	Lump-sum taxes (in real terms)
$C^r, C^p$	ROT and PIH consumption	с	Goods consumption
$L^r, L^p$	ROT and PIH labor supply	$X_{ii}$	Monetary policy shock
$\pi, \pi^{f}$	Overall domestic and foreign inflation	Ν	Net worth
$\pi_c$	Consumption good inflation	S	Nominal exchange rate

# **Description of Variables**

Table 1

Source: Self-prepared.

The money market interest rate Rn is the p.a. PRIBOR 3M interest rate for CR, whilst BRIBOR 3M (2006 – 2008) and EURIBOR 3M (since 2009) rates are used for SR. *Y*,  $\pi$  and *EX* data originate from International Monetary Fund database <a href="http://elibrary-data.imf.org">http://elibrary-data.imf.org</a>. *Rn* and housing sector inflation data were obtained from the CR's and SR's statistical offices.

Stationarity of the observed data series was verified using the KPSS test statistics. As a standard DSGE model feature, all observed and unobserved *steady state* variables are simulated within the model. *Parameter calibration* as shown in Table 2 was performed while keeping consistency with the data as well as general practices used for DSGE model construction. As an illustrative example, we discuss the calibration of the parameter  $\phi$  (net worth of housing ratio). For practical purposes, we use a macroeconomic-level approximation:  $LTV = 1 - \phi$ , where  $\phi$  is set to 0.7, taking into account both diverse individual-mortgage LTVs and the fact that a significant portion of housing stock owned by PIH households is not financed through housing loans (mortgages). Therefore, the LTV = 0.3 ratio as used in our DSGE model (benchmark) reflects the overall macroeconomic situation. Also, it may be reconciled to Lees (2009).

Table 2 Calibrated Parameters of the Model

Parameter	Calibrated value	Description of the calibrated parameter					
φ	0.7	Net worth ratio: $\phi = N/ah$					
$\Gamma_{d}$	0.52	Housing investment sensitivity to housing stock (see the TA for definition)					
ν	0.81	Steady state goods consumption as a proportion of overall consumption					
D A	0.9	Autocorrelation for technology shock					
P A P C	0.7	Autocorrelation of fiscal spending shock					
$\rho_{pf}$	0.8	Autocorrelation of foreign interest rate shock					
0 <sub>v</sub> f	0.8	Autocorrelation of foreign demand shock					
ργ	0.8	Autocorrelation of domestic interest rate shock					
$\theta$	0.5	$(1-\theta)$ is a probability of firm resetting its price					
α	0.65	Import weight in production function					
v	-0.2	Labor-imports substitution coefficient in production function					
$\delta_{h}$	-0.001	Reflects the cost of intermediation in the foreign currency bond market					
บิ	1	Export sensitivity to real exchange rate					
ζ	1	Export sensitivity to foreign demand					
$\phi_{B}$	0.33	Distribution of fiscal imbalances with respect to the government debt					
$\phi_{c}$	0.1	Distribution of fiscal imbalances with respect to the government exogenous spending					
$\dot{G}/Y$	0.2	Government spending/output ratio					
EX/Y	0.6	Exports/output ratio					
IM/Y	0.7	Imports/output ratio					

Source: Self-prepared using multiple sources, calibration process described in the TA.

*Prior information* processing and implementation as in Table 3 is illustrated using a few examples: monetary policy-related coefficients are established in accordance with DSGE models published by relevant authorities (Ministry of Finance, Central Bank). To our knowledge, the proportion of PIH consumers as given by the parameter n was not previously used in any DSGE model for CR or SR and we set it to 0.5. This may be compared to n = 0.7 used in a model for New Zealand where GDP per capita is higher (Lees, 2009). A complete and referenced description of parameter calibration and estimation priors is in the TA.

#### Table 3

Prior and Posterior Information on Parameters Estimated by the Model

Coefficients and their description	Prior mean & distr.		CR			SR		
Coefficients and their description			post.	90% conf. int.		post.	90% conf. int.	
$\eta$ : Consumer subst.: housing vs. goods	1.000	Ν	0.999	0.984	1.015	1.000	0.983	1.015
$\delta$ : Housing depreciation rate	0.005	В	0.010	0.005	0.015	0.010	0.006	0.015
<i>n</i> : Proportion of consumers that are PIH	0.500	В	0.651	0.536	0.791	0.643	0.512	0.770
$\gamma_{\pi}$ : Coeff. on CPI in monetary policy rule	1.500	Ν	1.495	1.479	1.511	1.495	1.477	1.510
$\gamma_{y}$ . Coeff. on $\tilde{y}$ in monetary policy rule	0.250	Ν	0.342	0.279	0.403	0.348	0.277	0.409
$\rho_i$ : Interest rate smoothing parameter	0.700	Ν	0.705	0.689	0.722	0.705	0.689	0.721
Ω: Sensitivity of interest rate premium to $\phi_t$	-0.100	Ν	-0.099	-0.116	-0.083	-0.097	-0.113	-0.082
$\phi_D$ : Sensitivity of dividend to net worth ratio	3.000	Ν	3.017	2.856	3.190	3.007	2.849	3.150
$\xi$ : Leisure coefficient in utility function	1.110	Ν	1.107	0.931	1.277	1.110	0.940	1.275
$\beta$ : Discount rate	0.990	В	0.990	0.988	0.992	0.990	0.988	0.992

*Note: N* and *B* in 3<sup>rd</sup> column stand for Normal and Beta distributions. Standard deviations are provided in TA. *Source:* Self-prepared using Dynare estimation outputs.

#### 4.2. Model Estimation and Impulse Response Functions

Fundamental maximum likelihood estimations (MLEs) are performed through iterative Kalman-filtering processes. As posterior distributions often have unknown distribution patterns, numerical random sample-generating techniques (such as the Metropolis-Hastings algorithm) are usually involved in DSGE estimation. Our model was estimated using the industry-standard approach: we use Dynare, a freeware add-on to the MATLAB software. Details on the Bayesian methodology used for our estimations may be observed from Koop (2003) and technical aspects of Dynare implementation are available from Adjemian (2012) and Griffoli (2010).

Model validation procedures were performed, taking into account the short observable time series available and the distortions in observed data that were due to the 2008 crisis onset. The so-called Business cycle moments' are included in Table 4, comparing first order autocorrelations AR(1) and pairwise correlations for the observed and model data. Observed moments are generally reconcilable to the model, yet individual differences exist and the model tends to underpredict AR(1) properties in some variables. Variance decomposition table (in the TA) shows that interest rate shocks  $\epsilon_{X_n}$  alone cause 51% – 86% of the

variance in long term predictions for q, Y,  $\pi$ , EX and Rn. Also, the combined effect of  $\epsilon_{X_n}$  and  $\epsilon_{R^f}$  constitutes nearly all the variance in long term predictions.

Model validation results are satisfactory, given the time period covered and the setup of our DSGE model, which we wanted to keep comprehensible for publication purposes. To summarize, the model (1) - (29) is a compromise between two contradictory goals: to make the model easy to understand and complete. Nevertheless, the DSGE model is a versatile concept. If necessary, different equations may be added or elaborated on (government and foreign sectors might be good candidates for such enhancement).

#### Table 4

Model Fit to Data Evaluated Through Business Cycle Moments

AR(1) Autocorrelations					Linear pairwise correlations (Pearson)						
AR(1)	CR	SR	Model	Corrs.	CR	SR	Model	Corrs.	CR	SR	Model
q	0.866	0.875	0.870	Y, q	0.591	0.775	0.474	$q, \pi_c$	0.637	0.421	0.557
Y	0.779	0.748	0.418	Y, EX	0.860	0.786	0.958	$q, R^n$	0.355	0.400	0.080
EX	0.754	0.747	0.484	$Y, R^n$	-0.013	0.293	-0.131	$EX, R^n$	0.030	0.254	-0.025
$R^n$	0.985	0.976	0.385	$Y, \pi_c$	0.344	0.232	0.939	EX, $\pi_c$	0.271	0.187	0.872
$\pi_c$	0.818	0.815	0.329	q, EX	0.511	0.603	0.278	$R^n, \pi_c$	0.453	0.238	-0.118

Source: Self-prepared using Dynare estimation outputs.

Impulse-response functions (IRFs) calculated from an estimated DSGE model are the expected paths of selected endogenous variables, conditional on a specified first period shock. In Graph 3, we use IRFs for *q* to illustrate the pro-volatile (pro-cyclical) LTV nature: higher LTV leads to higher absolute values of IRFs while lower LTV leads to IRFs with mitigated amplitudes of responses. Such behaviour and the pro-volatile nature of higher LTV values may be interpreted rather intuitively from the IRFs in Graph 3, where three alternative macro-economic levels of LTV are used: lower, benchmark and higher LTVs equal to 0.2; 0.3 and 0.4 respectively. Shocks corresponding to (30a), (30b) and (30d) were selected for Graph 3.

Given the *non-tradable nature of housing sector*, the humped IRF shapes for q following a technology shock  $\epsilon_A$  are attributable to the Balassa-Samuelson effect (Mandel and Tomšík, 2008, pp. 195 – 200), through increased productivity and GDP. Taylor rule based actions by the central authority and their effect on inflation may be used for describing the responses of q to  $\epsilon_{X_{ii}}$ . Inflation driven by government spending and crowding-out effect (to a lesser extent) would be shaping the IRFs of q following a  $\epsilon_G$  shock. While interpreting q, we need to bear in mind that real house prices (either expressed in y-o-y changes or as relative deviations from a steady state as used in our DSGE model) are the combination of two (potentially highly correlated) variables: nominal house prices and CPI inflation.

For all IRFs in Graph 3, we may observe that the modelled economic system is stable: within approximately 5 years (20 quarters) after the simulated shock, all IRFs for q exhibit a clear pattern of asymptotic return to the steady state.





*Note:* Legend shown for IRFs(SR): fiscal spending shock  $\rightarrow q$  applies to all IRFs in Graph 3. *Source:* Self-prepared using Dynare estimation outputs.

A complete set of LTV-scenario based IRFs describing the impact of all five shocks from (30a) to (30e) on q and Y is included in the TA. All IRFs for Y show that the impact of imposing experimental LTV values is very low for CR and SR

at the real GDP level where aggregated (non-sectorial) data are used. Such results correspond to the findings by Iacoviello and Neri (2010). Also, this may be interpreted in line with the relative weight of housing sector on GDP in both economies: the 2008 crisis was "imported" and neither CR's nor SR's overall macroeconomic dynamics or financial sectors were destabilized by *domestic* subprime mortgages with high individual LTV ratios.

At the same time, our results concerning the real GDP behaviour at the aggregated level do not undermine the suitability of the SOE DSGE model with housing sector as presented in this paper. Removing either the open economy or the housing sector features from the model would lead to substantial decrease in model performance on housing sector and aggregated data for CR and SR.

# 5. Scenario-based Forecasts and Other Applications of the Estimated Model

After estimating our model, evaluating its forecast performance and IRF dynamics, we may proceed to comment on selected conditional forecasts. As we dedicate this chapter to DSGE-based *applications*, we would like to refer the reader to Christoffel, Coenen and Warne (2010) for a complex review and illustrative examples of the DSGE forecasting *methodology*.

In this paper, we only focus on forecasting real house price dynamics. However, the model may be used for predicting future values of other real and nominal variables as well. Focusing on the impact of different LTV levels on the expected q dynamics, conditional forecasts were produced using alternative scenarios defined by macroeconomic LTV values. The benchmark LTV of 0.3 was experimentally lowered to 0.2 and 0.1 and increased values of 0.4 and 0.5 were used as well. Due to space limitations, Graph 4 only shows the results for LTV = 0.2 and 0.4 (a complete output is provided in the TA). For both economies, the overall conclusion is as follows: for LTV values from 0.1 to 0.4, lower LTVs lead to narrower confidence intervals for the predictions and increased LTVs lead to less reliable q predictions, (i.e. with wider confidence intervals) while the mean forecasts are not significantly influenced. Such results are consistent with the interpretation of IRFs provided in Section 4.2: higher LTVs are closely related with magnified reactions to any (exogenous) fluctuations in the DSGE model (1) – (29), therefore q and other variables become less predictable under higher LTVs. Again, this backs up the description of high LTV as a risk factor, a pro-volatile and cycle enhancing agent. At LTV = 0.5, forecasts diverge significantly from all other scenarios discussed here, presumably due to an overstated and unrealistic LTV parameter.

## Graph 4

Forecasts of q for CR and SR Based on Alternative LTV Settings





Other scenarios and conditional forecasts may be conveniently prepared: for example, a *government spending* scenario may be used to forecast q dynamics under strong fiscal expansion, e. g. simulated by setting the G/Y ratio to 0.3 instead of 0.2 (see Table 2) during the forecast period, other parameters kept unchanged with respect to the benchmark model. Similarly, an *exports scenario* may be produced by amending the benchmark DSGE model by setting the *EX/Y* parameter to 0.65 instead of 0.6 (ceteris paribus). This would mimic the influence of an expanding foreign economy (say, Germany) on a domestic SOE (CR or SR). Forecasts for both scenarios mentioned in this paragraph may be found in the TA.

Using the approach described above, we may produce conditional forecasts for selected variables, provided relevant macro- and microeconomic conditions can be expressed or approximated through fine-tuning the DSGE model parameters – either individually (ceteris paribus) or by inserting complex parameter-based scenarios. However, economical and mathematical plausibility of any such amendment must be closely observed because DSGE models (including ours) are not very robust against radical parameter editing.

Leaving aside parameter changes and conditional forecasts, other important applications for the DSGE model as per equations (1) - (29) may exist. Given its housing sector-specific properties, our model may be conveniently amended to encompass a generalized non-tradable goods sector. This would allow for studying the inflation differentials corresponding to the Balassa-Samuelson effect, as both CR and SR are still catching up economically with the "old" EU countries. Also, the non-tradable sector may be used for improved exchange rate pass-through analysis, i.e. for a stratified analysis of exchange rate shocks' transmission into domestic prices for tradables and non-tradables.

As a key part of this article, we disclose a complete and annotated Dynare code for our model in the TA. The code is available for the readers to replicate our estimates, experiment with parameter settings and to amend the model for additional task-specific purposes.

# Conclusions

We present a relatively compact, yet effective and versatile SOE DSGE model. The model is estimated for CR and SR and evaluated for consistency with observed data. Overall, our model performs well in the aspects addressed and may be successfully used for scenario based impulse response analysis, conditional forecasting and for other macroeconomic and housing sector-specific policy analyses for CR and SR.

We take advantage of the SOE features of our DSGE model in order to produce conditional ex-ante forecasts under different macroeconomic conditions, focusing mainly on alternative LTVs. Using IRFs calculated from the estimated DSGE model, for both CR and SR we find that higher LTV values have a significant pro-volatile impact on the housing sector. This finding is supported by scenario-based forecasts under different LTVs: with higher overall mortgage exposure, macroeconomic variables become more difficult to predict reliably.

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